**Subject Code: MATD01**

# GRAPH THEORY AND COMBINATORICS

**Course Outcomes:**

Upon completion of this course the student will be able to:

1. Identify types of graph, outline properties of graphs, describe when the graphs are said to be same even though the shapes are different (isomorphism) and apply to some practical problems like seven bridge problem, traveling sales man problem.
2. Describe how to color the vertices/ edges of a graph, apply graph coloring in map coloring, describe what is a tree and its properties and apply the concept of trees in constructing optimal prefix codes. Determine the shortest path between two vertices, write algorithms for finding minimal spanning trees and apply the concepts in transport network.
3. Apply the techniques of counting to identify the number of ways in which a given task can be accomplished without list all the possibilities explicitly.
4. Identify the different physical situations in which principle of inclusion and exclusion can be used for counting.
5. Derive the generating function for the given situation and evaluate the required coefficient. Solve the recurrence relation and interpret the solution.

## Graph Theory

### UNIT -I

**Introduction to Graph Theory**: Definitions and Examples, Subgraphs, Complements, and Graph Isomorphism, Vertex Degree, Euler Trails and Circuits, Planar Graphs, Hamilton Paths and Cycles.

### UNIT -II

**Introduction to Graph Theory contd**.: Graph Colouring, and Chromatic Polynomials.

**Trees:** Definitions, Properties, and Examples, Rooted Trees, Trees and Sorting, Weighted Trees and Prefix Codes. Minimal spanning Tree, Transport Networks: Max-Flow Min-cut Theorem, Matching theory.

## Combinatorics

### UNIT –III

**Fundamental Principles of Counting**: The Rules of Sum and Product, Permutations(linear, circular, identical objects), Combinations – The Binomial Theorem, Combinations with Repetition, The Catalon Numbers.

**The Principle of Inclusion and Exclusion**: The Principle of Inclusion and Exclusion,GeneralizationsofthePrinciple.

### UNIT -IV

Derangements – Nothing is in its Right Place, Rook Polynomials. **Generating Functions**: Introductory Examples, Definition and Examples – Calculational Techniques, Partitions of Integers. The Exponential Generating Function, The Summation Operator.

## UNIT -V

**Recurrence Relations:** First Order Linear Recurrence Relation, The Second Order Linear Homogeneous, Recurrence Relation with Constant Coefficients, The Non-homogeneous Recurrence Relation, The Method of Generating Functions.

**Text Books:**


1. Ralph P. Grimaldi, ”Discrete and Combinatorial Mathematics”, 5th Edition, *Pearson Education*, 2012. (Chapter 11, Chapter 12.1-12.4, Chapter 13, Chapter 1.1-1.4 , Chapter 8.1-8.4, Chapter 9.1 to 9.5 Chapter 10.1 -10.4).
2. Alan Tucker, “Applied Combinatorics”, 5th Edition, Wiley-India, 2011


**Reference Books:**


1. F. Harary, Graph Theory, Addison-Wesley Publishing Co., 1969.
2. Richard A. Brualdi, “Introductory Combinatorics”, 5th Edition, Pearson Prentice Hall, 2014.
3. Geir Agnarsson & Raymond Geenlaw, “Graph Theory Modeling, Applications, and Algorithms”, Pearson Prentice Hall, 2008.

**Subject Code: MATD02**

# ALGEBRAIC GRAPH THEORY

**Course Outcomes:**

Upon completion of this course the student will be able to:

1. Identify types of graph, outline properties of graphs, describe when the graphs are said to be same even though the shapes are different (isomorphism).
2. Describe how to color the vertices/ edges of a graph, apply graph coloring in map coloring, describe what is a tree and its properties and apply the concept of trees in constructing optimal prefix codes. Determine the shortest path between two vertices.
3. Determine the Vector spaces and subspaces, Null spaces, column spaces, linear transformations, linearly independent sets, bases, dimension of a vector space, rank, change of basis.
4. DetermineIncidence Matrix, Adjacency Matrix and their Spectrum of a graph.
5. Determine the Laplacian Matrix and computing Laplaciean Eigen values and their bounds of graphs and regular graphs

## UNIT -I

**Introduction to Graph Theory**: Definitions and Examples, Subgraphs, Complements, and Graph Isomorphism, Vertex Degree, Euler Trails and Circuits, Planar Graphs, Hamilton Paths and Cycles.

## UNIT -II

**Introduction to Graph Theory contd**.: Graph Coloring, and Chromatic Polynomials. **Trees:** Definitions, Properties, and Examples, Rooted Trees, Trees and Sorting, Weighted Trees and Prefix Codes.

## UNIT –III

**Vector spaces**: Vector spaces and subspaces, Null spaces, column spaces, linear transformations, linearly independent sets, bases, dimension of a vector space, rank, change of basis, Rank Nullity Theorem.

**Eigen values and Eigen vectors:** Introduction, characteristic equation, diagonalization, Eigen vectors and linear transformations.

## UNIT -IV

**Preliminaries**: Matrices, Trace and determinant, Vector spaces associated with a matrix,Minors,Nonsingular matrices,Orthogonally,Schurz complement, Inverse of a partitioned matrix,Eigenvalues of symmetric matrices,Positive definite matrices,Interlacing for eigenvalues,

**Incidence Matrix:** Rank,Minors,Path matrix,Integer generalized inverses,Moore–Penrose inverse.

**Adjacency Matrix:**Eigenvalues of some graphs,Determinant,Bounds,Energy of a graph,AntiNonsingular trees.

## UNIT -V

**Laplacian Matrix:** Basic properties, Computing Laplacian eigenvalues, Matrix-tree theorem, Bounds for Laplacian spectral radius.

**Regular Graphs:** Perron–Frobenius theory,Adjacency algebra of a regular graph, strongly regular graphs and friendship theorem, Graphs with maximum energy.

**Text Books:**


1. Ralph P. Grimaldi, Discrete and Combinatorial Mathematics, 5th Edition, Pearson Education, 2012.
2. R.B. Bapat, Graphs and Matrices, Hindustan Book Agency, New Delhi, Springer, 2013.
3. David C. Lay, Linear algebra and its applications, 3rd Edition, Pearson Education, 2003.


**Reference Books:**


1. F. Harary, Graph Theory, Addison-Wesley Publishing Co., 1969.
2. Norman Biggs, "Algebraic Graph Theory" First South Asia Edition, Cambridge University Press, 2016.
3. Kenneth Hoffman and Ray Kunze, Linear Algebra, 2nd Edition, Pearson Education, 2010.

**Subject Code: MATD05**

# ADVANCED NUMBER THEORY

## UNIT-1

Partitions - partitions of numbers, the generating function of p(n), other generating functions, two theorems of Euler, Jacobi's triple product identity and its applications.

## UNIT-2

1ψ1 - summation formula and its applications, combinatorial proofs of Euler's identity, Euler's pentagonal number theorem, Franklin's combinatorial proof

## UNIT-3

Congruence properties of partition function, the Rogers - Ramanujan Identities.

## UNIT-4

Ramanujan's general theta-function and its particular cases, Theta-function identities of Ramanujan found in his Chapter 16 of his second notebook, Quintuple product identity and its applications.

## UNIT-5

Ramanujan's cubic continued fractions, Rogers-Ramanujan continued fractions and related theta-function identities.

**Text Books:**

1. G. H. Hardy and E. M. Wright (1979), An Introduction to Theory of Numbers, Oxford University Press, 5$^{th}$Ed.
2. B. C. Berndt, Ramanujan's Notebooks, Part III, Springer-Verlag, New York, 1991.

**Reference Books:**

1. G. E. Andrews – The Theory of Partitions, Addison Wesley, 1976.
2. B. C. Berndt, Number Theory in the Spirit of Ramanujan, American Mathematical Society, Providence, RI 2006.

**Subject Code: MATD03**

# ADVANCED GRAPH THEORY

**Course Outcomes:**

Upon completion of this course the student will be able to:

1. Study spanning trees and Partitions.
2. Discuss and understand the importance of the concepts of Coverings and independence.
3. Distinguish between Eulerian graphs, Hamiltonian graphs and apply to the problems of Traversibility.
4. Understand Plane, planar graphs and Euler's beautiful formula.
5. Apply theory of matrices to Graph theory.

## UNIT-I

**Trees and Partitions:** Characterization of trees, centers and centroids, spanning trees and partitions.

## UNIT-II

**Connectivity and line:** Connectivity, Menger's theorem. Coverings and Independence. Critical points and lines.

## UNIT-III

**Traversibility:** Eulerian graphs, Hamiltonian graphs, Line graphs - Some properties of line graphs. Characterization of line graphs, special line graphs, Line graphs and traversibility.

## UNIT-I V

**Factorization Planarity:** Plane and planar graphs, Euler's formula, Characterizations of planar graphs, Nonplanar graphs, outerplanar graphs.

## UNIT- V

**Colorability:** The chromatic number, Five color theorem, Matrices-The adjacency matrix, The incidence matrix, The cycle matrix.

**Text Books:**


1. F. Harary-Graph Theory, Addition Wesley Reading Mass, 1969.
2. K.R. Parthasarathy-Basic Graph Theory, Tata McGraw-Hill, New Delhi, 1994.


**Reference Books:**


1. N. Deo-Graph Theory With Applications to engineering and Computer Science, Prentice Hall of India, 1987.
2. G. Chartand and L.Lesniak-Graphs and Diagraphs, Qwadsworth and Brooks, 2ndEd.

**Subject Code: MATD04**

## COMPLEX ANALYSIS

**Course Outcomes:**

Upon completion of this course the student will be able to:

1. Analyse analytic functions and Exponential functions.
2. Describe Cauchy integral formula and local properties of analytic functions.
3. Recognise and analyse definite integrals.
4. Continue to develop proof techniques.
5. Explain concepts from Univalent functions.

**UNIT 1**

Analytic function, Cauchy-Riemann equations, Elementary properties of Harmonic functions, Polynomials and Rational functions, Green's function.

**UNIT 2**

Conformal Mapping, Complex integration, Morera's theorem, Liouville's theorem, Fundamental theorem of Algebra. Maximum modulus principle, Schwarz Lemma, Taylor series, Laurent series, Zeros and poles of a function, Meromorphic functions.

**UNIT 3**

The residue at a singularity, Residue theorem, the argument principle, Rouche's theorem, contour integration and its applications to improper integrals, evaluation of a real integrals, improper integrals involving sines and cosines, definite integrals involving sines and cosines.

**UNIT 4**

Subordination, Basic principles, Simple examples for differential and integral operators, Coefficient inequalities.

**UNIT 5**

Univalent functions, Area theorem, Growth theorem, Distortion theorems, Bieberbach theorem, Koebe's one quarter theorem, Functions with positive real part, Starlike and convex functions, Alexander theorem, Close to convex functions and spirallike functions, some coefficient theorems.

**Text Books:**


1. L. V. Ahlfors , Complex Analysis, 2nd Edition, McGraw Hill Book Co , New York , (1996).
2. P. L. Duren, Univalent Functions, Springer-Verlag, (1983).
3. G. S. Goodman, Univalent Functions, Vol-I and II, Mariner Publishing Co., Tanga, Florida, (1983).


**Reference Books:**


1. R. P. Boas, Invitation to Complex Analysis, The Random House, (1987).
2. B. C. Palka, An Introduction to Complex Function Theory, Springer, (1991).
3. S. Ponnusamy, Foundations of Complex Analysis, Narosa, (1995).
4. Armugam, Tangapandi, Isoc, Somasundaram, Complex Analysis, Satech Publications Pvt. Ltd.